

# Storage of spin squeezing in a two-component Bose-Einstein condensate

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Efficient control of spin squeezing in a two-component Bose-Einstein Condensate is studied by rapidly turning-off the external field at a time that maximal spin squeezing appears. We show that strong reduction of spin fluctuation can be maintained in a nearly fixed direction for a long time. We explain the underlying physics unambiguously, and present analytical expressions of the maximal-squeezing time.

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Spin squeezing has attracted much attention for decades not only because of fundamental physical interests [1, 2, 3, 4, 5], but also for its possible application in atomic clocks for reducing quantum noise [2] and quantum information [6, 7, 8, 9, 10]. Formally, the spin squeezing is quantified via a parameter  $\xi = (\Delta \hat{J}_{\mathbf{n}})_{\min} / \sqrt{j/2}$ , where  $(\Delta \hat{J}_{\mathbf{n}})_{\min}$  represents the smallest variance of a spin component  $\hat{J}_{\mathbf{n}} = \hat{J} \cdot \mathbf{n}$  normal to the mean spin  $\langle \hat{J} \rangle$ . For the coherent spin state (CSS), the variance  $(\Delta \hat{J}_{\mathbf{n}})_{\min} = \sqrt{j/2}$  (i.e.,  $\xi = 1$ ). A state is called spin squeezed state (SSS) if its variance of the spin component is smaller than that of the CSS, i.e.  $\xi < 1$ .

Kitagawa and Ueda have investigated the spin squeezing generated by the so-called one-axis twisting (OAT) model with Hamiltonian:  $\hat{H}_{\text{OAT}} = 2\kappa \hat{J}_z^2$  [1]. Possible realization of the OAT-type spin squeezing in a two-component Bose-Einstein Condensate (TBEC) has been proposed in Refs. [6, 11]. Sørensen et al. also argued that macroscopic quantum entanglement can be characterized by using the OAT-type spin squeezing in the TBEC [6]. Besides the TBEC, most recently, Takeuchi et al. considered another realization of the OAT-type spin squeezing by using the interactions between atoms and off-resonant light (paramagnetic Faraday rotation) [12]. To coherently control spin squeezing, Law et al. introduced additional Josephson-like (or Raman) coupling to the OAT model:  $\hat{H}_R = 2\kappa \hat{J}_z^2 + \Omega_R \hat{J}_x$  [13]. Such a model has also been used to prepare arbitrary Dicke states [14].

In addition to the generation of the SSS itself, it is desirable to maintain not only the squeezing but also its direction for a long time [7]. Jaksch et al. have shown that the OAT-type SSS can be stored for arbitrarily long time by removing the self-interaction [15]. However, it might not be easy to handle in experiment since the precisely designed additional pulses are crucially required. In this letter, we propose a simple mechanism to obtain long-lasting spin squeezing in the TBEC. Our scheme is quite easy to realize in experiment since it can be achieved by turning-off the Josephson coupling once the TBEC reaches its maximal spin squeezing.

We consider a two-component weakly-interacting BEC [16, 17] consisting of  $N$  atoms in different atomic hyperfine states  $|a\rangle$  and  $|b\rangle$  coupled by a time-varying mi-

crowave field with Rabi frequency  $\Omega_{rf}$ . Based on the two-mode approximation [18, 19, 20, 21, 22, 23], the total Hamiltonian can be described by ( $\hbar = 1$ ):

$$\hat{H}(t) = 2\kappa \hat{J}_z^2 + \Omega(t) \hat{J}_x, \quad (1)$$

where  $\kappa = (\kappa_{aa} + \kappa_{bb} - 2\kappa_{ab})/4$ , and  $\kappa_{\alpha\beta} = g_{\alpha\beta} \int d^3\mathbf{r} |\phi_{\alpha}(\mathbf{r})\phi_{\beta}(\mathbf{r})|^2$ , with  $g_{\alpha\beta} = 4\pi a_{\alpha\beta}/m$  ( $\alpha, \beta = a, b$ ) being the  $s$ -wave scattering strengths between atoms. The condensate-mode functions  $\phi_{\alpha}$  normalized to unity satisfies a coupled Gross-Pitaevskii equations [24]. Here we focus on the case that the external field is turned off rapidly at a certain time  $t_0$ , so the time-dependent Josephson-like coupling can be written as  $\Omega(t) = \Omega_R \Theta(t_0 - t)$ , where  $\Omega_R = \Omega_{rf} \int d^3\mathbf{r} \phi_a^*(\mathbf{r})\phi_b(\mathbf{r})$  and  $\Theta(t)$  is the step function.

The state vector at arbitrary time  $t$  can be expanded in terms of eigenstates of  $\hat{J}_z$ :  $|\psi(t)\rangle = \sum_m c_m(t) |j, m\rangle$ , where  $-j \leq m \leq j$  and  $j = N/2$ . The equations of motion for the amplitudes  $c_m(t)$  are obtained by solving time-dependent Schrödinger equation. We consider an initial CSS  $|j, -j\rangle_x = e^{-i\pi J_y/2} |j, -j\rangle$ , then the initial amplitudes  $c_m(0) = \frac{(-1)^{j+m}}{2^j} \binom{2j}{j+m}$ . Since the initial CSS satisfies  $c_{-m}(0) = c_m(0)$  for even  $N$ , and  $c_{-m}(0) = -c_m(0)$  for odd  $N$ , we can prove the mean spin always along the  $x$  direction. In addition, we will consider only positive  $\kappa$  case by assuming  $a_{aa} + a_{bb} > 2a_{ab}$ . However, our results keep valid in the opposite case by using initial maximum weight state of  $\hat{J}_x$ , i.e.,  $|j, j\rangle_x$ .

Now let us briefly explain the basic principle of our scheme. The initial CSS can be prepared by applying a short  $\pi/2$  pulse to a single-component BEC with all the atoms being in the internal state  $|a\rangle$  [6, 23]. After that, the external Josephson field is immediately switched on, so dynamical evolution of the spin system is governed by the Hamiltonian (1) with  $\Omega(t) = \Omega_R$ . If the coupling is optimally chosen, the Josephson interaction results in an enhanced spin squeezing compared with that of the OAT [13]. For  $N = 10^3$ , we find that the maximal squeezing  $\xi_0 = 8.7076 \times 10^{-2}$  can be obtained by choosing arbitrary  $\Omega_R$  in a region  $10.777 \leq \Omega_R/\kappa \leq 10.818$ . As shown by the dashed lines of Fig. 1, the squeezing  $\xi$  and the mean spin  $\langle \hat{J}_x \rangle$  show collapsed oscillations [25]. At the time

$t_0$ ,  $\xi$  decreases to its local minimum  $\xi_0$  with  $\theta_{\min} = 0$ , while  $\langle \hat{J}_x \rangle$  increases to its maximal value  $\langle \hat{J}_x \rangle_0$ . The basic features of our scheme are exhibited by the solid lines of Fig. 1. We find that if we turn off the Josephson field at the time  $t_0$ , the maximal squeezing  $\xi_0$  can be stored in a fixed direction (i.e.,  $\theta_{\min} = 0$ ) for a long time.

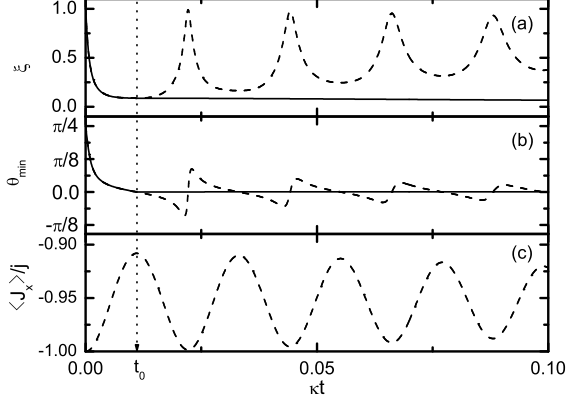


FIG. 1: Time evolution of (a) the squeezing parameter, (b) the squeezing angle, and (c) the mean spin  $\langle \hat{J}_x \rangle$  for  $N = 10^3$  and  $\Omega_R = 10.8\kappa$ . Dashed lines: constant-coupling case; Solid lines: turning-off external field at the time  $\kappa t_0 = 1.1 \times 10^{-2}$ , with its position indicated by the vertical dotted line.

To understand our above observations, we investigate probability distribution of the spin state,  $|c_m|^2 = |\langle j, m | \psi(t) \rangle|^2$ . As shown in the insets of Fig. 2, we find that compared with the initial CSS, the maximally SSS at  $t_0$  has a very sharp probability distribution centered at the lowest spin projection, i.e.,  $m = 0$  (for even  $N$ ) or  $m = \pm 1/2$  (for odd  $N$ ). Such a sharp probability distribution of the SSS can be explained qualitatively by considering the familiar phase model [26]. By replacing  $\hat{J}_z \rightarrow P_\Phi = -i\partial_{\hat{\Phi}}$  and  $\hat{J}_x \rightarrow (N \cos \hat{\Phi})/2$ , with  $\hat{\Phi}$  being macroscopic phase difference between two condensate components, one obtain

$$H_\Phi = -2\kappa \frac{\partial^2}{\partial \hat{\Phi}^2} + \frac{\Omega_R N}{2} \cos \hat{\Phi}, \quad (2)$$

where we have taken  $\Omega(t) = \Omega_R$  to simulate quantum dynamics of the spin system before turning-off the field. The phase model Hamiltonian allows us to regard the spin system as a fictitious particle with effective mass  $(4\kappa)^{-1}$  subject to a pendulum potential. Moreover, the particle behaves as a pendulum rotating with frequency  $\omega_{\text{eff}} = \sqrt{2\kappa\Omega_R N}$  in phase space  $(\Phi, P_\Phi)$ , where mean phase difference  $\Phi = \langle \hat{\Phi} \rangle$ . As shown in Fig. 2(a) and (b), starting from vertical distributed initial points ( $\Phi = \pi$  and  $P_\Phi = -j, -j+1, \dots, j$ ), one obtains the distribution elongated horizontally at the time  $t \simeq T/4$ , where  $T = 2\pi/\omega_{\text{eff}}$  is the period of the pendulum. It is reasonable to assume that the projection of the distribution along  $\Phi$  will always be symmetrical to  $\pi$ , e.g.,  $\Phi_0 = 2\pi - \Phi_0$  and  $\langle \hat{J}_x \rangle_0 = j \cos(\Phi_0)$ , as Fig. 2(b).

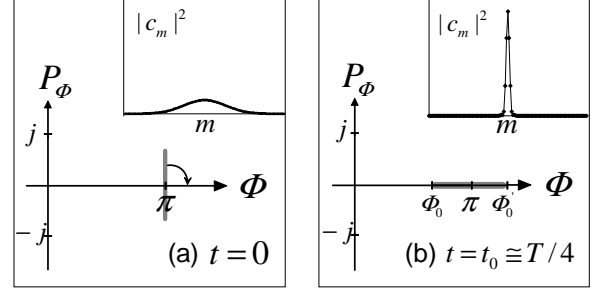


FIG. 2: Schematic picture of the probability distribution in phase space  $(\Phi, P_\Phi)$  for: (a) the initial CSS, (b) the SSS at  $t \simeq T/4$ . The insets: the corresponding distribution  $|c_m|^2$  as a function of  $m$  (or  $P_\Phi$ ) obtained numerically.

Based upon the phase model, we explain intuitively why the obtained SSS has a sharp distribution with the lowest spin projection being occupied predominantly. In addition, we find that the sharp distribution accompanies with the maximal mean spin, i.e.,  $\langle \hat{J}_x \rangle$ :  $-j \rightarrow \langle \hat{J}_x \rangle_0$  as  $\Phi : \pi \rightarrow \Phi_0$ , thus  $(d\langle \hat{J}_x \rangle / dt)_{t_0} \equiv 0$ . On the other hand, from Heisenberg equation of  $\hat{J}_x$  we know the relation  $d\langle \hat{J}_x \rangle / dt \sim \langle \hat{J}_z \hat{J}_y + \hat{J}_y \hat{J}_z \rangle \sim A \tan(2\theta_{\min})$  [for  $A \neq 0$ , see the deduction of Eq. (6)], so we further obtain  $\theta_{\min} = 0$  at  $t_0$ , as shown in Fig. 1(b). More important, we obtain analytical expression of the maximal-squeezing time

$$\kappa t_0 \simeq \kappa \frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{\kappa}{2\Omega_R N}}, \quad (3)$$

which is valid for large  $N$  ( $\geq 10^3$ ) and small coupling with  $\kappa < \Omega_R \ll N\kappa$ . By comparing with exact numerical results, we find that Eq. (3) gives accurate prediction of the maximal-squeezing time for  $\Omega_R$  near or larger than the optimal coupling. Law et al. have investigated the optimal coupling as a function of  $N$  based on a wide range of numerical simulations [13], from which we suppose that the optimal coupling obeys power rule  $\Omega_R/\kappa \sim N^{1/3}$  for the large  $N$ . Such as  $N = 10^3$ , the optimal coupling is about  $\Omega_R = 10.8\kappa$  and Eq. (3) gives  $\kappa t_0 = 1.069 \times 10^{-2}$ , consistent with numerical result  $1.104 \times 10^{-2}$ . Note that Eq. (3) predicts the time scale for the sharp distribution of the SSS and the maximal  $\langle \hat{J}_x \rangle$  (also  $\theta_{\min} = 0$ ), which, however, does not necessarily correspond to the maximal squeezing (see below).

If the Josephson field is turned off at  $t_0$ , the spin system is governed only by the self-interaction Hamiltonian  $2\kappa \hat{J}_z^2$ , so the distribution  $|c_m(t)|^2$  is conserved while the relative phases among the spin projections are subject to change. When the SSS at the time  $t_0$  exhibits a very sharp distribution in  $m$ , as shown in the inset of Fig. 2(b), the effect of relative phases induced by the self-interaction gives negligible influence to the squeezing. To show this, we suppose the SSS at  $t_0$  takes the

form

$$|\psi(t_0)\rangle = \frac{e^{i\varphi} \sin \alpha}{\sqrt{2}} (|j, 1\rangle + |j, -1\rangle) + \cos \alpha |j, 0\rangle, \quad (4)$$

for even  $N$  case, or

$$|\psi(t_0)\rangle = \frac{e^{i\varphi} \sin \alpha}{\sqrt{2}} (|j, 3/2\rangle - |j, -3/2\rangle) + \frac{\cos \alpha}{\sqrt{2}} (|j, 1/2\rangle - |j, -1/2\rangle), \quad (5)$$

for the odd  $N$ , where  $\varphi$  and  $\alpha$  represent the relative phase and the amplitude, respectively. Fig. 3 shows the squeezing parameter  $\xi$  as a function of  $\alpha$  and  $\varphi$ , where two distinct features are observed. Firstly,  $\xi$  is minimized as  $\alpha \rightarrow 0$  [2], which implies that the maximal squeezing occurs as long as the SSS has a very sharp distribution with a large amplitude of the lowest spin projection or, equivalently, as the SSS approaches to the ground-state of  $2\kappa\hat{J}_z^2$ . Secondly,  $\xi$  looks insensitive to the relative phase  $\varphi$  for the SSS with a sharp distribution. Since the self-interaction only varies the relative phase  $\varphi$  with a fixed  $\alpha$ , Fig. 3 explains qualitatively the storage of the maximal squeezing after  $t_0$ .

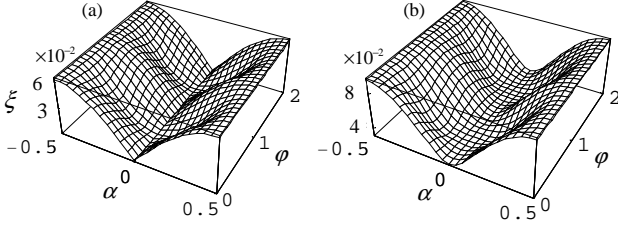


FIG. 3: The squeezing parameter  $\xi$  as a function of  $\alpha$  and  $\varphi$  (in units of  $\pi$ ) for (a) the even number case ( $N = 1000$ ); (b) the odd number case ( $N = 1001$ ), calculated by using Eqs. (4) and (5), respectively.

The validity of Eqs. (4) and (5) can be tested by considering two exact solvable cases with  $N = 2$  and  $N = 3$ . Taking the optimal coupling  $\Omega_R = \kappa$  for the two-atom system and  $\Omega_R = 2\kappa$  for the three-atom one, we obtain the maximally-squeezed states  $|\psi(t_n)\rangle = i(-1)^{n+1}e^{-i\kappa t_n}|j = 1, m = 0\rangle$  [2] and  $|\psi(t_n)\rangle = \frac{i(-1)^n}{\sqrt{2}}e^{-3i\kappa t_n/2}(|3/2, 1/2\rangle - |3/2, -1/2\rangle)$ , respectively, where  $t_n = (2n + 1)\pi/S$  for any integer  $n$ , and the level spacing  $S = 2\sqrt{\Omega_R^2 + \kappa^2}$  for  $N = 2$  and  $S = 2\sqrt{\Omega_R^2 + 2\kappa\Omega_R + 4\kappa^2}$  for  $N = 3$ . Obviously, the spin state  $|\psi(t_n)\rangle$  is the ground state of the self-interaction Hamiltonian  $2\kappa\hat{J}_z^2$ , which results in exactly constant  $\xi$  with zero  $\theta_{\min}$  by rapid turning-off the external field at the times  $t_n$ . For large  $N$ , the SSS at the time  $t_0$  no longer corresponds to the ground state of  $2\kappa\hat{J}_z^2$ , but approaches to it compared with the initial CSS. Consequently, almost constant  $\xi$  can be achieved after turning-off the external field.

The storage itself in our scheme does not depend on  $\Omega_R$ , and the optimal coupling is chosen here to store the maximal squeezing  $\xi_0$ . If  $\Omega_R$  is smaller than the optimal coupling, there exist two time scales:  $t_0$  for the vanishing  $\theta_{\min}$ , and  $\tau_M$  for the maximal squeezing. Eq. (3) still works well to give the time scale of  $\theta_{\min} = 0$ , but fails to predict that of the maximal squeezing. As shown in Fig. 4(a), for  $\Omega_R = 5\kappa$  real maximal squeezing occurs at  $\kappa t_0 = 6.915 \times 10^{-3}$ . From Fig. 4(a) we also find that the maximal  $\langle \hat{J}_x \rangle$  and  $\theta_{\min} = 0$  appears at the same time  $\kappa t_0 = 1.687 \times 10^{-2}$ , at which the probability distribution of the SSS is sharp enough [see the inset of Fig. 4(b)]. As a result, a less squeezed variance with  $\theta_{\min} = 0$  can be stored by turning-off the external field at the time  $t_0$  [see the red and the green lines of Fig. 4(a)].

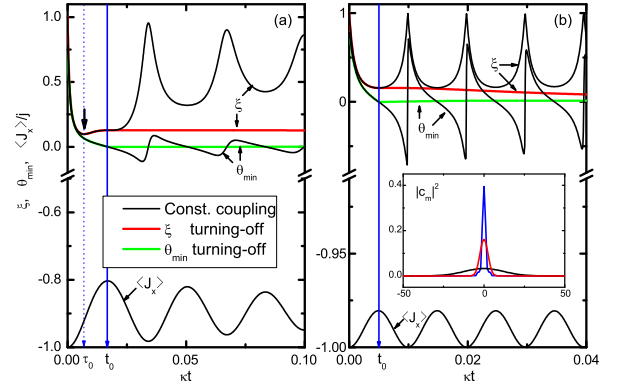


FIG. 4: (Color online) Time evolution of  $\xi$ ,  $\theta_{\min}$  and the mean spin  $\langle \hat{J}_x \rangle/j$  for  $N = 10^3$ , (a)  $\Omega_R = 5\kappa$ , and (b)  $\Omega_R = 50\kappa$ . Black curves in (a) and (b) are the corresponding quantities for the constant-coupling case. Red (green) curves represents the squeezing  $\xi$  ( $\theta_{\min}$ ) after turning off the field at time  $t_0$ . Inset: Probability distribution  $|c_m|^2$  as a function of  $m$  for the SSS at the times  $\tau_0$  (the black line), and  $t_0$  with the blue line for (a) and the red line for (b).

It is worth mentioning that the above qualitative explanation can not be applied to the SSS with a broad probability distribution. In fact, typical OAT-scheme [1] relies solely on the evolution of relative phases induced by the self-interaction, where the initial CSS shows a broad probability distribution. As shown by the red line of Fig. 4(b), for a large coupling  $\Omega_R = 50\kappa$ , the squeezing parameter decreases slightly after turning-off the external coupling. This is because the SSS at  $t_0$  exhibits a relatively broad probability distribution [see the inset of Fig. 4(b)]. From the green line of Fig. 4(b), we find that the reduced variance is stored in the fixed direction with  $\theta_{\min} = 0$ . By comparing the numerical and the analytical result of  $t_0$  for  $\Omega_R = 50\kappa$ , we also find that the numerical result  $4.945 \times 10^{-3}\kappa^{-1}$  agrees very well with  $4.967 \times 10^{-3}\kappa^{-1}$  estimated from Eq. (3).

Before closing, we prove that the mean spin always appears in the  $x$  direction, and explain why we study the spin squeezing in the small-coupling regime. Note

that the linear combinations of the probability amplitudes  $p_m^{(\pm)}(t) = c_m(t) \pm c_{-m}(t)$  obey two closed sets of first-order ordinary differential equations. For even  $N$ , the fact that all  $p_m^{(-)}(0) = 0$  results in  $p_m^{(-)}(t) = 0$ , namely  $c_{-m}(t) = c_m(t)$ . On the other hands, for odd  $N$  all  $p_m^{(+)}(t)$  are zero, i.e.,  $c_{-m}(t) = -c_m(t)$ . Since  $c_{-m}(t) = \pm c_m(t)$ , we obtain simple expressions:  $\langle \hat{J}_y \rangle = \langle \hat{J}_z \rangle = 0$ , and  $\langle \hat{J}_x \rangle \neq 0$ , i.e., the mean spin is along the  $x$  direction. Consequently, the spin component normal to the mean spin reads  $\hat{J}_n = \hat{J}_y \sin \theta + \hat{J}_z \cos \theta$ . By minimizing the variance  $(\Delta \hat{J}_n)^2$  with respect to  $\theta$ , we obtain the squeezing angle  $\theta_{\min} = \frac{1}{2} \tan^{-1}(B/A)$  and  $(\Delta \hat{J}_n)_{\min}^2 = \frac{1}{2}C - \frac{1}{2}\sqrt{A^2 + B^2}$ , where  $A = \langle \hat{J}_z^2 - \hat{J}_y^2 \rangle$ ,  $B = \langle \hat{J}_z \hat{J}_y + \hat{J}_y \hat{J}_z \rangle$ , and  $C = \langle \hat{J}_z^2 + \hat{J}_y^2 \rangle$ . From Heisenberg equations of motion of the spin  $\hat{J}_\alpha$  for  $\alpha = x, y, z$ , one can obtain formal solutions for the constant coupling case:  $C = j(j+1) - \langle \hat{J}_x^2 \rangle$ ,  $A = -C + j(1 - \Omega_R/\kappa) - (\Omega_R/\kappa)\langle \hat{J}_x \rangle$ , and  $B = -(2\kappa)^{-1}d\langle \hat{J}_x \rangle/dt$ . Note that for  $B = 0$  and  $A \neq 0$ , the spin squeezing takes place along  $z$  axis (i.e.  $\theta_{\min} = 0$ ) with the corresponding squeezing parameter

$$\xi_0^2 = 1 - (\Omega_R/\kappa) \left[ 1 + \langle \hat{J}_x \rangle_0 / j \right], \quad (6)$$

where  $\langle \hat{J}_x \rangle_0$  is the maximum value of the mean spin. For an extremely strong coupling ( $\Omega_R \gg \kappa N$ ),  $\langle \hat{J}_x \rangle_0 \rightarrow -j$  and  $\xi_0 \rightarrow 1$  so the squeezing becomes very weak. This is the reason why we discuss the spin squeezing in the small-coupling regime ( $\kappa < \Omega_R \ll N\kappa$ ).

Finally, we estimate several important parameters for experimental realization. Following Ref. [6], we consider  $^{23}\text{Na}$  atoms in the hyperfine states  $|F = 1, M_F = \pm 1\rangle$  trapped in a spherically symmetric potential  $V_a = V_b = m\omega^2 r^2/2$ . The self-interaction strength can be solved by

applying the Thomas-Fermi approximation, yielding

$$\kappa \simeq \frac{15^{2/5} \hbar \omega}{14} \frac{a_{\text{eff}}}{a_{\text{ho}}} \left( \frac{a_{\text{ho}}}{N a_{aa}} \right)^{3/5}, \quad (7)$$

where  $a_{\text{ho}} = \sqrt{\hbar/m\omega}$  being the harmonic oscillator length, and  $a_{\text{eff}} = a_{aa} + a_{bb} - 2a_{ab}$  effective scattering length. For  $^{23}\text{Na}$  atoms, we take  $a_{aa} = a_{bb} = 2a_{ab}$  [6] and  $a_{\text{eff}} = a_{aa} = 2.75\text{nm}$  [17], then the self-interaction strength  $\kappa \simeq 3.2448 \times 10^{-4} \hbar \omega$ . For the case  $N = 10^3$  and  $\Omega_R = 10.8\kappa$ , we have obtained  $t_0 = 1.1041 \times 10^{-2} / (\hbar^{-1} \kappa) = 34.03 \omega^{-1}$ , which corresponds to the maximal-squeezing time about 10.83 ms for  $\omega = 2\pi \times 500\text{Hz}$ .

In summary, we have investigated coherent control of spin squeezing by turning-off Josephson field at the maximal-squeezing time. We show that by applying the optimal coupling then turning off later, the maximal squeezing can be stored in the  $z$  axis for a long time, which can be explained in terms of the probability distribution of the SSS. For a sharp distribution with a large amplitude of the lowest spin projection, the effect of the self-interaction gives small contribution to the squeezing and its direction. We find the analytic expression of the maximal-squeezing time by considering the phase model. Our scheme for the storage of spin squeezing is quite robust for a wide range of Josephson coupling.

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 [1] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).  
 [2] D. J. Wineland *et al.*, Phys. Rev. A **46**, R6797 (1992); *ibid.* **50**, 67 (1994).  
 [3] A. Kuzmich *et al.*, Phys. Rev. Lett. **79**, 4782 (1997).  
 [4] J. Hald *et al.*, Phys. Rev. Lett. **83**, 1319 (1999).  
 [5] J. M. Geremia *et al.*, Science **304**, 270 (2004).  
 [6] A. Sørensen *et al.*, Nature (London) **409**, 63 (2001).  
 [7] K. Helmerson and L. You, Phys. Rev. Lett. **87**, 170402 (2001); M. Zhang *et al.*, Phys. Rev. A **68**, 043622 (2003).  
 [8] X. Wang and B. C. Sanders, Phys. Rev. A **68**, 012101 (2003).  
 [9] J. K. Korbicz *et al.*, Phys. Rev. Lett. **95**, 120502 (2005).  
 [10] S. Yi and H. Pu, Phys. Rev. A **73**, 023602 (2006).  
 [11] U. V. Poulsen and K. Molmer, Phys. Rev. A **64**, 013616 (2001).  
 [12] M. Takeuchi *et al.*, Phys. Rev. Lett. **94**, 023003 (2005).  
 [13] C.K. Law *et al.*, Phys. Rev. A **63**, 055601 (2001).

[14] S. Raghavan *et al.*, Opt. Commu. **188**, 149 (2001).  
 [15] D. Jaksch *et al.*, Phys. Rev. A **65**, 033625 (2002).  
 [16] D. S. Hall *et al.*, Phys. Rev. Lett. **81**, 1539 (1998); *ibid.* **81**, 1543 (1998).  
 [17] J. Stenger *et al.*, Nature **396**, 345 (1998).  
 [18] G. J. Milburn *et al.*, Phys. Rev. A **55**, 4318 (1997).  
 [19] A. Smerzi *et al.*, Phys. Rev. Lett. **79**, 4950 (1997).  
 [20] J.I. Cirac *et al.*, Phys. Rev. A **57**, 1208 (1998).  
 [21] M.J. Steel and M.J. Collett, Phys. Rev. A **57**, 2920 (1998).  
 [22] E. M. Wright *et al.*, Phys. Rev. Lett. **77**, 2158 (1996).  
 [23] D. Gordon and C. M. Savage, Phys. Rev. A **59**, 4623 (1999).  
 [24] A. Sinatra and Y. Castin, Eur. Phys. J. D **8**, 319 (2000).  
 [25] G. S. Agarwal and R. R. Puri, Phys. Rev. A **39**, 2969 (1989); G. R. Jin and W. M. Liu, *ibid.* **70**, 013803 (2004).  
 [26] D. Jaksch *et al.*, Phys. Rev. Lett. **86**, 4733 (2001); C. Menotti *et al.*, Phys. Rev. A **63**, 023601 (2001); A. Micheli *et al.*, *ibid.* **67**, 013607 (2003).